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A Lifshitz black hole in four dimensional R^2 gravity

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ABSTRACT: We consider a higher derivative gravity theory in four dimensions with a negative cosmological constant and show that vacuum solutions of both Lifshitz type and Schrödinger type with arbitrary dynamical exponent z exist in this system. Then we find an analytic black hole solution which asymptotes to the vacuum Lifshitz solution with $z = 3/2$ at a specific value of the coupling constant. We analyze the thermodynamic behavior of this black hole and find that the black hole has zero entropy while non-zero temperature, which is very similar to the case of BTZ black holes in new massive gravity at a specific coupling. In addition, we find that the three dimensional Lifshitz black hole recently found by E. Ayon-Beato *et al.* has a negative entropy and mass when the Newton constant is taken to be positive.

KEYWORDS: Gauge-gravity correspondence, Black Holes

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Contents

1	Introduction	1
2	The Lifshitz black hole solution	2
2.1	Vacuum solutions	3
2.2	The Lifshitz black hole solution	3
2.3	Unusual thermal properties	5
2.4	Remarks on the unusual thermodynamic behavior	6
3	Conclusion and discussion	7

1 Introduction

The anti-de Sitter/conformal field theory correspondence (AdS/CFT) [1] provides an elegant way to study strongly coupled quantum field theories by relating them to certain classical gravity theories. This holographic method is also quite useful in describing condensed matter systems, for some nice reviews, see [2–5] and references therein. However, some condensed matter systems realized in laboratories at their critical points are described by non-relativistic conformal field theories (NRCFT). Thus it would be very useful to study the non-relativistic version of AdS/CFT to gain more knowledge of the strong coupling behavior of such condensed matter systems.

Non-relativistic conformal symmetry contains the scaling symmetry

$$t \rightarrow \lambda^z t, \quad \mathbf{x} \rightarrow \lambda \mathbf{x}, \quad (1.1)$$

where z is the so-called dynamical exponent. For $z = 1$ this scaling symmetry comes back to the familiar relativistic scale invariance. Such a non-relativistic scale invariance (1.1) can be exhibited by either a Galilean-invariant theory or a Lifshitz-invariant theory. In [6, 7], the gravity dual for the Schrödinger type field theory was proposed, while in [8], the gravity dual for the Lifshitz type field theory was proposed.

The thermal version of gauge/gravity dual is very useful in realistic applications, so it is very interesting to heat up the previous Schrödinger vacuum and Lifshitz vacuum solutions proposed in [6–8]. It is easy to heat up the Schrödinger vacuum and embed the thermal solutions into string theory [9–11], while, however, it is very difficult to find black hole solutions which are dual to the thermal Lifshitz field theory [12–14]. Some interesting attempts have been made in [15, 16]. Other interesting discussions on the gravity dual for Lifshitz field theory could be found in [17–25].

Recently, in [26] an interesting analytic black hole solution which asymptotes to the Lifshitz vacuum solution was found in the framework of the three dimensional New Massive

Gravity (NMG) [27] at a specific value of coupling. In NMG, a special higher derivative gravitational term was added to the usual action of general relativity and many interesting things may happen at certain specific couplings [28–45]. This motivates us to look for higher dimensional asymptotically Lifshitz black holes in pure higher derivative gravity system. At a specific value of coupling, we indeed found an analytic asymptotically Lifshitz black hole in four dimensions with the dynamical exponent being $z = 3/2$ and this solution could be viewed as a generalization of the three dimensional case [26] to four dimensions. However, we did not find any analytic Lifshitz black hole solutions for the higher derivative gravity in five dimensions.

The black hole solution we found has very unusual thermodynamic properties. Using the Wald entropy formula, we find that it has a zero entropy while with a nonzero temperature which depends on the mass parameter in the solution. This is very analogous to the circumstance of the BTZ black hole in NMG at a specific coupling [29]. Though this does not necessarily mean that the dual field theory is trivial, it is still mysterious in the framework of pure gravity theory that a black hole has zero entropy while with non-zero temperature.

In the next section, we give the analytic Lifshitz black hole solution and the specific higher derivative gravity theory. In section 3, we discuss the thermodynamic properties of this black hole and compare it with the BTZ black hole in NMG at a specific coupling. In the last section, we give our conclusion and discussions.

2 The Lifshitz black hole solution

Motivated by [26] where an analytic asymptotically Lifshitz black hole solution was found in NMG [27], which is a higher derivative gravity system in three dimensions, we add higher derivative terms to the four dimensional Einstein gravity theory and consider the following action

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R - 2\lambda + \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu}), \quad (2.1)$$

where α, β are coupling constants, G is the four dimensional Newton constant and λ is the cosmological constant. As we know in four dimensions the Gauss-Bonnet term is a topological invariant and does not affect the equation of motion, so this action (2.1) is the most general form for gravity theories with R^2 high derivative terms in four dimensions. The corresponding equation of motion for the action (2.1) is

$$G_{\mu\nu} + \lambda g_{\mu\nu} + Y_{\mu\nu} = 0, \quad (2.2)$$

where

$$\begin{aligned} G_{\mu\nu} &= R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \\ Y_{\mu\nu} &= (2\alpha + \beta)(g_{\mu\nu} \nabla^2 - \nabla_\mu \nabla_\nu) R + \beta \nabla^2 G_{\mu\nu} \\ &\quad + 2\alpha R \left(R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R \right) + 2\beta (R_{\mu\rho\nu\sigma} - \frac{1}{4} g_{\mu\nu} R_{\rho\sigma}) R^{\rho\sigma}. \end{aligned} \quad (2.3)$$

In the next subsections we will present Lifshitz and Schrödinger vacuum solutions and Lifshitz black hole solutions for this action.

2.1 Vacuum solutions

We assume that the Lifshitz vacuum solution is of the form

$$ds^2 = -\frac{r^{2z}}{\ell^{2z}}dt^2 + \frac{\ell^2}{r^2}dr^2 + \frac{r^2}{\ell^2}(dx^2 + dy^2), \quad (2.4)$$

where z is the dynamical exponent. We substitute this assumption (2.4) into the equation of motion (2.2) and find that the vacuum Lifshitz solution with the form (2.4) exists only when the coupling constants and the cosmological constant satisfy the following relation

$$\begin{aligned} \beta\ell^{-2} &= \frac{1 - 4\alpha\ell^{-2}(z^2 + 2z + 3)}{2(2 + z^2)}, \\ \lambda\ell^2 &= -\frac{1}{2}(z^2 + 2z + 3). \end{aligned} \quad (2.5)$$

From the relation (2.5) we can easily see that to make sure that z is a real number, the cosmological constant should satisfy the condition $\lambda\ell^2 < -1$. In addition, we can have the ordinary AdS vacuum solution with $z = 1$ when $4\alpha\ell^{-2} + \beta\ell^{-2} = 1/6$ and $\lambda\ell^2 = -3$.

The isometry group of this vacuum solution is generated by [8, 17]

$$M_{ij} = -i(x^i\partial_j - x^j\partial_i), \quad P_i = -i\partial_i, \quad H = -i\partial_t, \quad D = -i(zt\partial_t + x^i\partial_i + r\partial_r) \quad (2.6)$$

which constitutes the Lifshitz symmetry algebra. The momentum P_i , Hamiltonian H and angular momentum M_{ij} enjoy the usual commutators, while the dilatation operator has nonvanishing commutators with the other generators as $[D, P_i] = iP_i$, $[D, H] = iH$, $[D, M_{ij}] = i(2 - z)M_{ij}$. It's natural to conjecture that the quantum gravity of (2.1) on the background (2.4) is dual to a 2+1 dimensional non-relativistic quantum field theory which has a Lifshitz scale invariance with dynamical exponent z .

Interestingly the action (2.1) also has a Schrödinger vacuum solution

$$ds^2 = -\frac{r^{2z}}{\ell^{2z}}dt^2 + \frac{\ell^2}{r^2}dr^2 + \frac{r^2}{\ell^2}(-2tdtdx + dy^2), \quad (2.7)$$

when

$$\begin{aligned} \beta\ell^{-2} &= \frac{24\alpha\ell^{-2} - 1}{2(2z^2 - z - 4)}, \\ \lambda\ell^2 &= -3. \end{aligned} \quad (2.8)$$

Some analytic Schrödinger black hole solutions are available in the literatures [9–11]. We will focus our attention on finding Lifshitz black hole solutions for the action (2.1) in the next subsection.

2.2 The Lifshitz black hole solution

Following [26], we assume that the metric of the asymptotically Lifshitz black hole solution has the following form

$$ds^2 = -\frac{r^{2z}}{\ell^{2z}}F(r)dt^2 + \frac{\ell^2}{r^2}H(r)dr^2 + \frac{r^2}{\ell^2}(dx^2 + dy^2), \quad (2.9)$$

where $F(r)$ and $H(r)$ are functions depending on the radial coordinate r only. In order for the black hole solution to be asymptotic to the Lifshitz vacuum solution (2.4), we demand that these functions obey $\lim_{r \rightarrow \infty} F(r) = \lim_{r \rightarrow \infty} H^{-1}(r) = 1$. To make it a black hole solution, we demand that $F(r)$ and $H(r)$ vanish at a given radius $r = r_H$ where the horizon is located.

The equation of motion turns out to be solved by

$$F(r) = H^{-1}(r) = 1 - \frac{r_H^3}{r^3}, \quad z = \frac{3}{2} \quad (2.10)$$

when the coupling constants and the cosmological constant take the following value

$$\alpha \ell^{-2} = 1/33, \quad \beta \ell^{-2} = 0, \quad \lambda \ell^2 = -33/8. \quad (2.11)$$

Here this solution is valid only at $z = 3/2$ and at other values of the dynamical exponent z , we did not find any analytic solutions.

Then the static asymptotically Lifshitz black hole for $z = 3/2$ is given by

$$ds^2 = -\frac{r^3}{\ell^3} \left(1 - \frac{r_H^3}{r^3}\right) dt^2 + \frac{\ell^2}{r^2} \frac{dr^2}{1 - \frac{r_H^3}{r^3}} + \frac{r^2}{\ell^2} (dx^2 + dy^2), \quad (2.12)$$

and the corresponding gravity action is

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R - 2\lambda + \alpha R^2), \quad (2.13)$$

where $\alpha = \ell^2/33$ and $\lambda = -33/8\ell^2$.

It can be easily checked that the solution (2.12) has a curvature singularity at $r = 0$, where some scalar combinations of the Riemann tensor diverge. The metric has a horizon at $r = r_H$, and the boundary is localized at $r \rightarrow \infty$. From the geometrical point of view, it is indeed a black hole solution. In addition, this black hole is asymptotically to the Lifshitz vacuum solution (2.4) with $z = 3/2$. It is natural to conjecture that the quantum gravity of (2.1) with the above special parameters on the background (2.12) is dual to a $2 + 1$ dimensional thermal non-relativistic Lifshitz quantum field theory with dynamical exponent $z = 3/2$. It is expected that this solution can also describe the quantum critical region in condensed matter systems at nonzero temperature.

Before discussing the thermal properties of this black hole, let us first have a look at the action (2.13) at the specific coupling we take here. At the specific coupling (2.11) the trace of the equation of motion (2.2) gives

$$R = 4\lambda + 6\alpha \nabla^2 R. \quad (2.14)$$

This is always satisfied by constant curvature metrics with $R = 4\lambda$. Then by substituting $R = 4\lambda$ back to the equation of motion (2.2) we find that the equation of motion vanishes, which means that all constant curvature solutions with $R = 4\lambda$ are solutions of the equation of motion (2.2) at the specific coupling (2.11). It is easy to check that the black hole solution we found is just a constant curvature solution of this kind.

2.3 Unusual thermal properties

Now let us focus on the thermal properties of the Lifshitz black hole (2.12). The Hawking temperature of the black hole is easy to find

$$T = \frac{3}{4\pi r_H} \left(\frac{r_H}{\ell}\right)^{5/2}. \quad (2.15)$$

By Wald's formula for the black hole entropy [46],

$$S = -2\pi \int_H \frac{\partial \mathcal{L}}{\partial R_{abcd}} \epsilon_{ab} \epsilon_{cd}, \quad (2.16)$$

where \mathcal{L} is the Lagrangian of a gravity theory, we find that the Lifshitz black hole solution (2.12) has a vanishing entropy in the action (2.13). The vanishing entropy is closely related to the fact that our solution satisfies the relation: $1 + 2\alpha R = 0$.

Indeed, all constant curvature solutions with $R = 4\lambda$ are solutions to the action (2.13) and our solution is a constant curvature solution with $R = -1/2\alpha = 4\lambda$. Furthermore, we find that the action (2.13) is always vanishing for our solution, even when a boundary term is added

$$S_{\text{bt}} = -\frac{1}{8\pi} \int_{\partial\mathcal{M}} d^3x \sqrt{-h} (1 + 2\alpha R) K, \quad (2.17)$$

where K is the extrinsic curvature for the boundary hypersurface $\partial\mathcal{M}$ with induced metric h . This implies that both the free energy and mass of the black hole vanish as well

$$F = M = 0, \quad (2.18)$$

although the black hole has a nonvanishing horizon radius r_H .

Such thermodynamic behavior looks very strange. However, we find that similar strange thermodynamic behavior also occurs for the three dimensional Lifshitz black hole found in [26]. The action of NMG in three dimensions is

$$I = \frac{1}{16\pi G} \int d^3x \sqrt{-g} \left[R - 2\lambda_0 - \frac{1}{m^2} K \right], \quad (2.19)$$

where

$$K = R^{\mu\nu} R_{\mu\nu} - \frac{3}{8} R^2, \quad (2.20)$$

m is the mass parameter of this massive gravity and λ_0 is a constant which is different from the cosmological constant. The black hole solution present in [26] is

$$ds^2 = -\frac{r^{2z}}{\ell^{2z}} F(r) dt^2 + \frac{\ell^2}{r^2} H(r) dr^2 + \frac{r^2}{\ell^2} dx^2, \quad (2.21)$$

where

$$F(r) = H^{-1}(r) = 1 - \frac{r_H^2}{r^2}, \quad (2.22)$$

$z = 3$, $m^2 = -1/(2\ell^2)$, $\lambda_0 = -13/(2\ell^2)$ and r_H is an integration constant. The black hole has a Hawking temperature

$$T = \frac{1}{2\pi\ell} \left(\frac{r_H}{\ell}\right)^3,$$

but a negative entropy

$$S = -\frac{A}{G},$$

by the Wald's formula, where $A = r_H L/\ell$ and L is the length of the coordinate x . By using the first law of black hole thermodynamics, $dM = TdS$, we find that the mass of the black hole (2.21) is negative

$$M = -\frac{L}{8\pi G\ell} \left(\frac{r_H}{\ell}\right)^4, \tag{2.23}$$

if taking the solution (2.21) with $r_H = 0$ as a vacuum solution being of vanishing mass.

Here we should note that if we take the three dimensional Newton constant G to be negative as in Topological massive gravity and NMG in asymptotically Minkowski spacetime, the entropy and the mass can be both positive. However, in usual gravity calculations concerned with black holes, we always take G to be positive, so the thermal behavior of this three dimensional black hole is quite unusual compared with ordinary black holes.

2.4 Remarks on the unusual thermodynamic behavior

It seems quite unphysical that a black hole solution possesses a zero entropy with arbitrary nonzero temperature, which is a property of thermal Minkowski vacuum or thermal AdS spacetime, or at least this may indicate that the corresponding field theory is trivial in some sense. However, in fact, such properties for black hole have also been found before in NMG for BTZ black holes, which does not necessarily mean that the corresponding field theory is trivial [28–30]. Let us have a look at what happened there.

The action (2.19) is the one for NMG in three dimensions and the BTZ black hole is a solution to this action. The central charges of NMG in the background of AdS_3 are [29]

$$c_L = c_R = \frac{3\ell}{2G} \left(1 - \frac{1}{2m^2\ell^2}\right). \tag{2.24}$$

The entropy of the BTZ black hole is proportional to the central charges and also has a factor $1 - 1/2m^2\ell^2$. Thus at a specific coupling $m^2\ell^2 = 1/2$, the entropy of the BTZ black hole vanishes while the temperature of the black hole is nonzero. This is very analogous to the condition we encounter here. In the case of NMG, it is shown that at the specific coupling $m^2\ell^2 = 1/2$ under Brown-Henneaux boundary conditions, the dual field theory is trivial. However, at this specific coupling under a relaxed log boundary condition, it is shown that the dual field theory is a log conformal field theory which is not trivial though the BTZ black hole has a zero entropy. On the gravity side this may indicate that there are other gravity solutions besides the BTZ black hole which satisfy the boundary conditions, such as pp wave solutions [35]. In the case of this asymptotically Lifshitz black hole, it is expected that there may be other gravity solutions which asymptotes the Lifshitz vacuum solution under certain boundary conditions and the whole system with all asymptotically Lifshitz solutions can describe the dual condensed matter system at the quantum critical region.

3 Conclusion and discussion

In this paper we considered a higher derivative gravity system (2.1) in four dimensions with a negative cosmological constant. We showed that vacuum solutions with both Lifshitz type (2.4) and Schrödinger type (2.7) isometry with arbitrary dynamical exponent z exist in this action. We also constructed an analytic black hole solution (2.12) which asymptotes to the vacuum Lifshitz solution with $z = 3/2$ at a specific value of coupling. However, we found that this black hole has an unusual thermodynamic behavior that it has zero entropy but non-zero temperature.

This may seem unphysical or at least indicate that the dual field theory may be trivial. However, we have an analogy of BTZ black holes in NMG in three dimensions, where at a specific coupling, the entropy of the BTZ black hole vanishes while the temperature is non-zero. In that case, the dual field theory can be trivial or nontrivial depending on the boundary conditions though the entropy is always zero at that specific coupling. Thus in this asymptotically Lifshitz case, we hope that there may also be a proper boundary condition under which the dual non-relativistic field theory is not trivial and can describe the condensed matter systems at the quantum critical region. Of course this still needs further investigation to see whether there exist other asymptotically Lifshitz solutions or whether the dual field theory is trivial.

Despite that the dual field theory may not be trivial, it is still mysterious in the framework of a gravity theory that a black hole can have zero entropy while non-zero temperature. Future work should be done to probe this black hole using various ways to get more information of this black hole or to find the origin of this unusual behavior.

In addition, we found that the three dimensional Lifshitz black hole presented recently in [26] has a positive Hawking temperature, but a negative entropy and a negative mass when we take the Newton constant to be positive. From the point of view of black hole thermodynamics, it is of great interest to further understand those unusual thermodynamic properties of black holes.

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